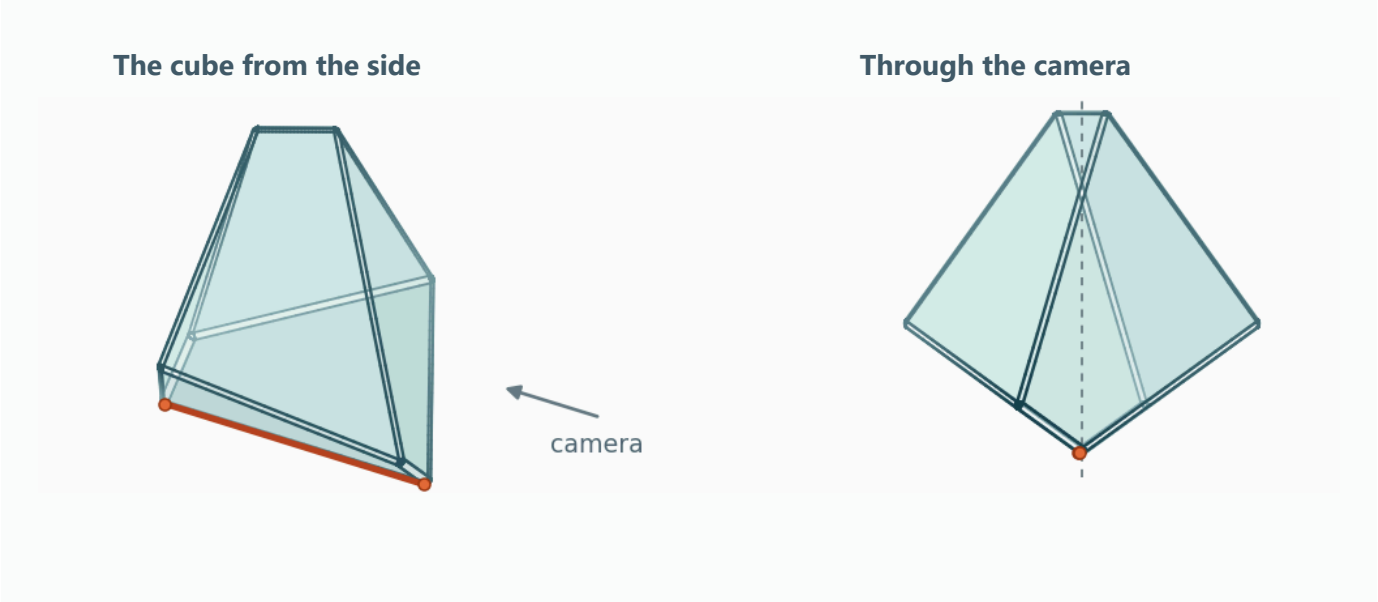
The shadows of a chiral cube

How a handed solid can cast a mirror-symmetric outline

A left hand and its mirror image have opposite handedness. A chiral solid has the same property in three dimensions. Its shadow can still have a fold line because the camera discards the depth that distinguishes front from back.



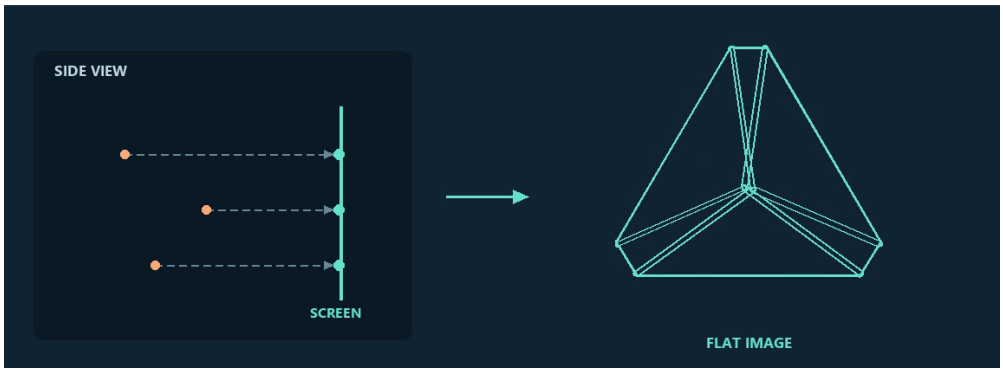
At an 80° turn, the orange edge measures 31.95 mm in space. Its ends appear only 0.13 mm apart in the projection, giving the outline a pointed base.

Three findings from the supplied STEP geometry

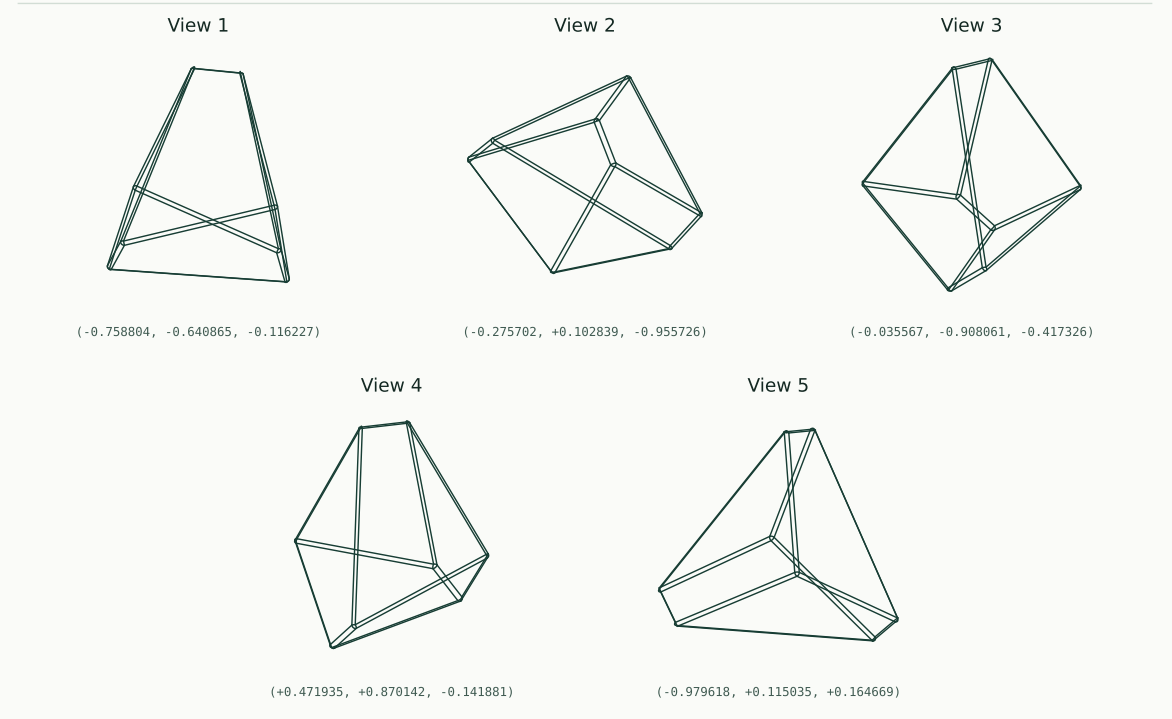
- Infinitely many viewing directions give a mirrored outline within 0.01 mm.
- All 51 CAD edges can match within 0.0078 mm. Exact equality is impossible for the complete wireframe at every viewing angle.
- The smallest orthographic silhouette has area 573.95 mm^2.

The solid and the projection

An orthographic camera sends parallel rays perpendicular to a flat screen. Each point keeps its horizontal and vertical position. Two points at different depths can land on the same spot. [1]



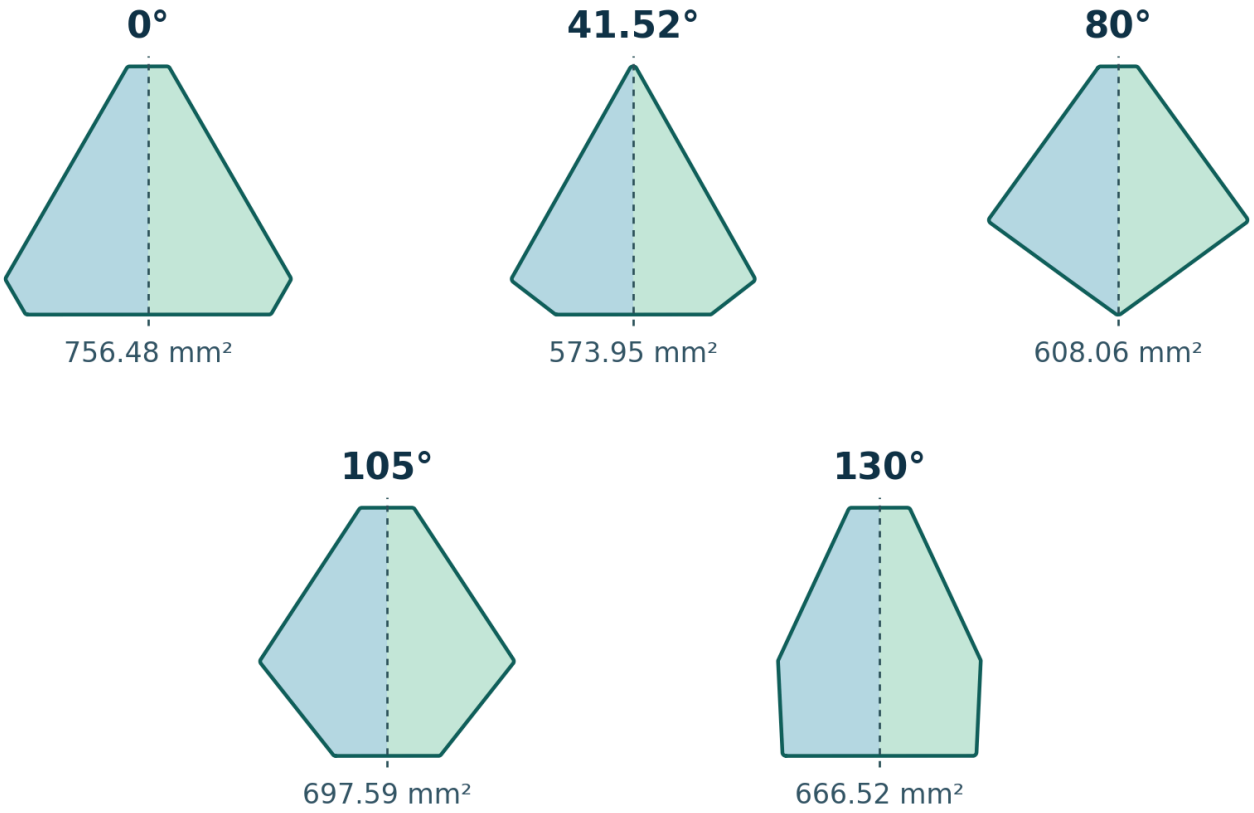
Parallel rays at left explain the flat construction. All edges of the rounded model appear in the projection at right, including those on the far side.



Five randomly chosen views of the chiral cube, at a common scale. Each triplet gives the unit direction from the object to the viewer. These illustrate typical orientations.

Five silhouettes from one family

Choose a nearly exact half-turn axis of the cube as screen vertical. Keep the sightline perpendicular to that axis while moving the camera. The dashed line in each drawing marks the same upright fold.



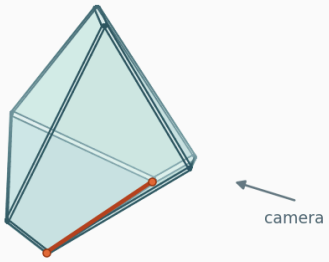
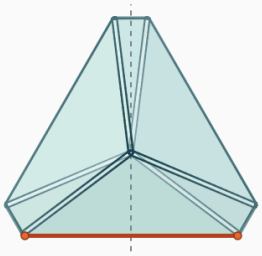
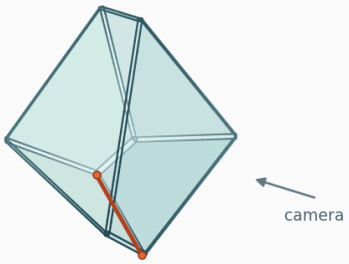
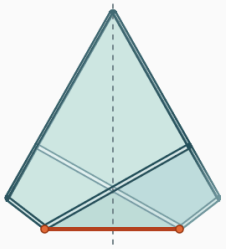
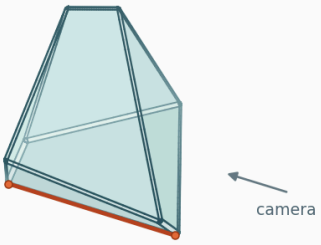
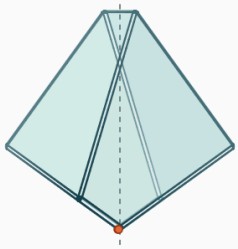
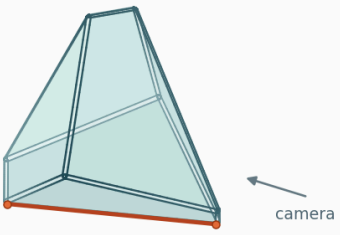
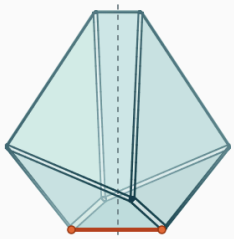
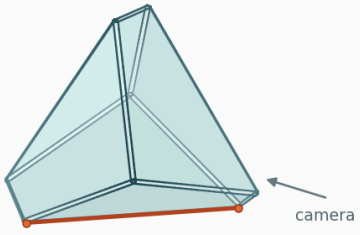
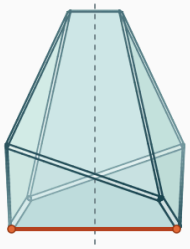
Why the fold survives a camera turn

A half-turn preserves height, swaps left with right, and swaps front with back. Removing depth leaves a left–right reflection. For this rounded model the eight corner centers match their half-turned partners to within 0.00000060 mm. Projection, taking the convex hull and adding the 0.5 mm rounding preserve that upper bound.

The same argument holds for every intermediate camera angle. The exact count for the literal outer contour at zero tolerance remains open here.

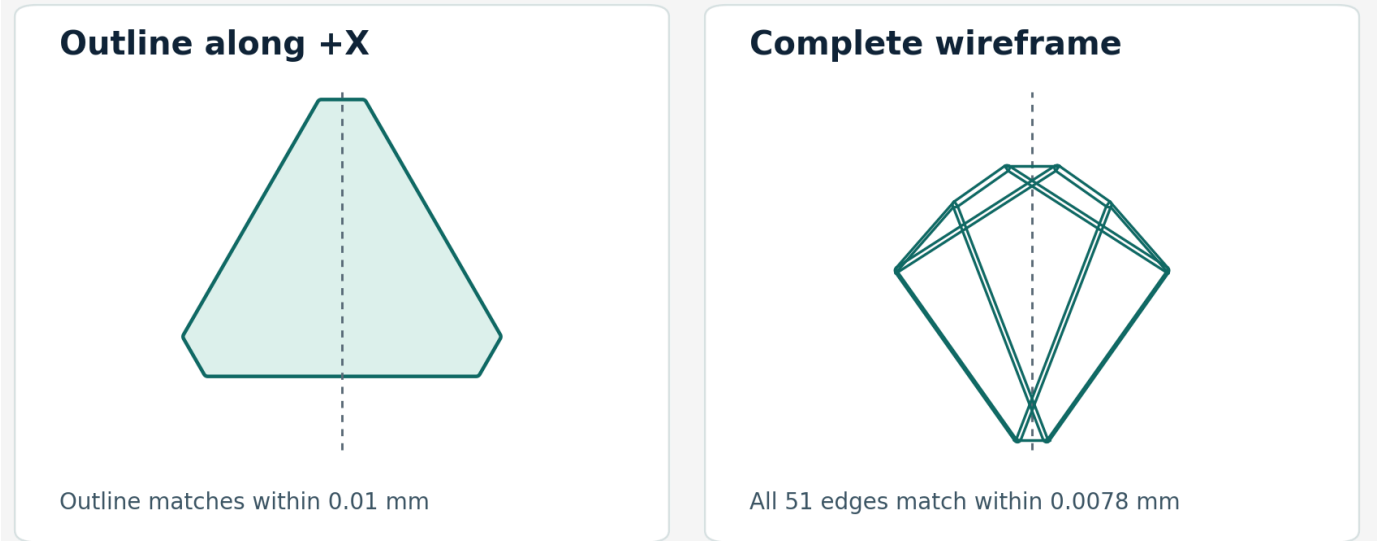
Five see-through orientations

The tilted view reveals depth. The fixed camera records the view on the right. The orange edge is the same physical edge in every row.

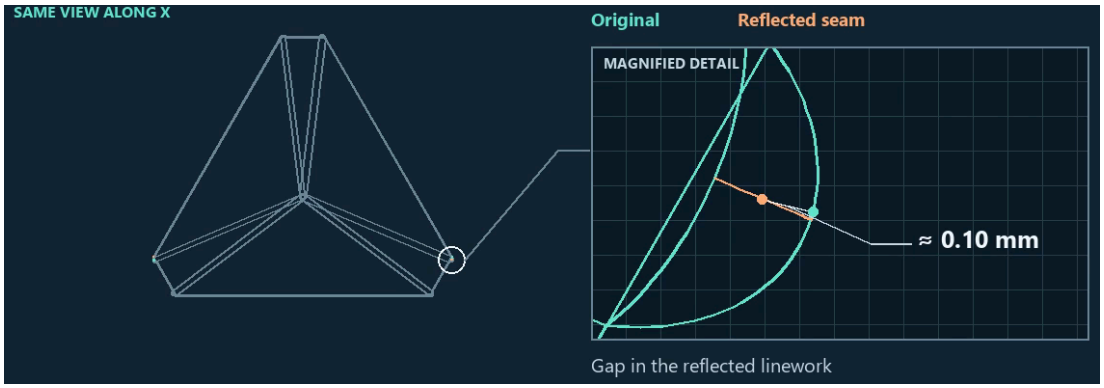
Cube turn	Tilted view	Camera view
0° 31.49 mm edge span		
41.52° 19.98 mm edge span		
80° 0.13 mm edge span		
105° 13.39 mm edge span		
130° 24.40 mm edge span		

The seams in a transparent drawing

A silhouette contains the outer boundary. A transparent wireframe adds the front and back edges, including lines a CAD system uses to close a curved surface. Those surface seams can change which parts of a drawing match.



At left, the +X silhouette folds within 0.000001 mm. At right, another direction brings the entire 51-edge wireframe within 0.0078 mm of its reflection.



Enlargement of the +X wireframe. The reflected seam misses the closest original line by about 0.104 mm. This gap belongs to this particular view.

The STEP file declares 0.01 mm of geometric uncertainty. All gap values here describe the recorded CAD model.

How to check infinitely many orientations

The exact test follows the centers of the circular edges. A tilted circle projects to an ellipse. Reflecting a nondegenerate elliptical arc also reflects its center. Each of the eight corner groups supplies a nondegenerate arc from every viewing direction.



1 Pair the centers

When the projected centers are separate, a mirror leaves each center on the fold or swaps it with another. There are 764 such assignments for eight centers. Partners must have equal screen heights, and their midpoints must all lie on one fold line. Exact rational bounds exclude every assignment for the recorded coordinates.

2 Check where centers can overlap

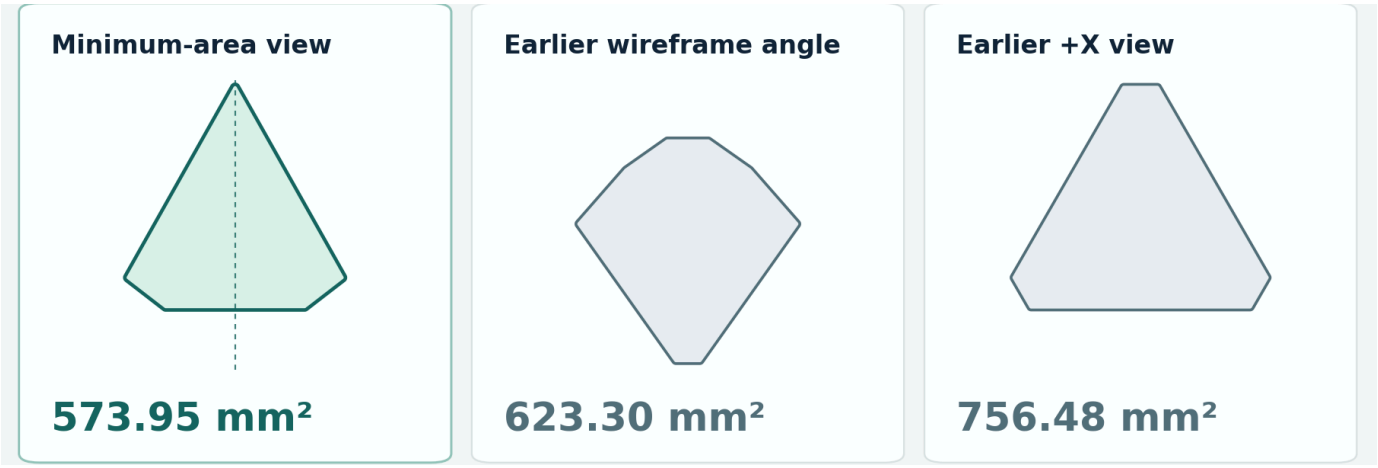
Two centers coincide on screen when the sightline runs along the line joining them. Eight centers give 28 pairs. The certificate also checks a small neighborhood of each direction to account for tiny differences among the stored centers. Projected distances force multiple noncollinear points to lie on one fold, which is impossible.

No viewing direction gives a perfect mirror of all 51 recorded edges. The near-match on page 5 and the family of silhouettes on page 3 satisfy different geometric tests.

Mirror gap means the largest distance from a point in either drawing to the nearest point in the other after reflection.

The smallest shadow

Here “smallest” means least area enclosed by the outer contour. The minimizing sightline is parallel to the intersection of two of the model’s supporting planes, about 41.52° from +X. Viewing from the opposite direction gives the same area.



The minimum is 7.9% smaller than the earlier angled wireframe view and 24.1% smaller than the +X view. The three silhouettes share one scale.

Accounting for the rounded corners

Project the eight core vertices and take their convex hull. An outward offset by radius r adds a strip along each polygon edge. The circular corner sectors together make one disk. If A is the core shadow area and P its perimeter, the rounded shadow area is $A + rP + \pi r^2$.

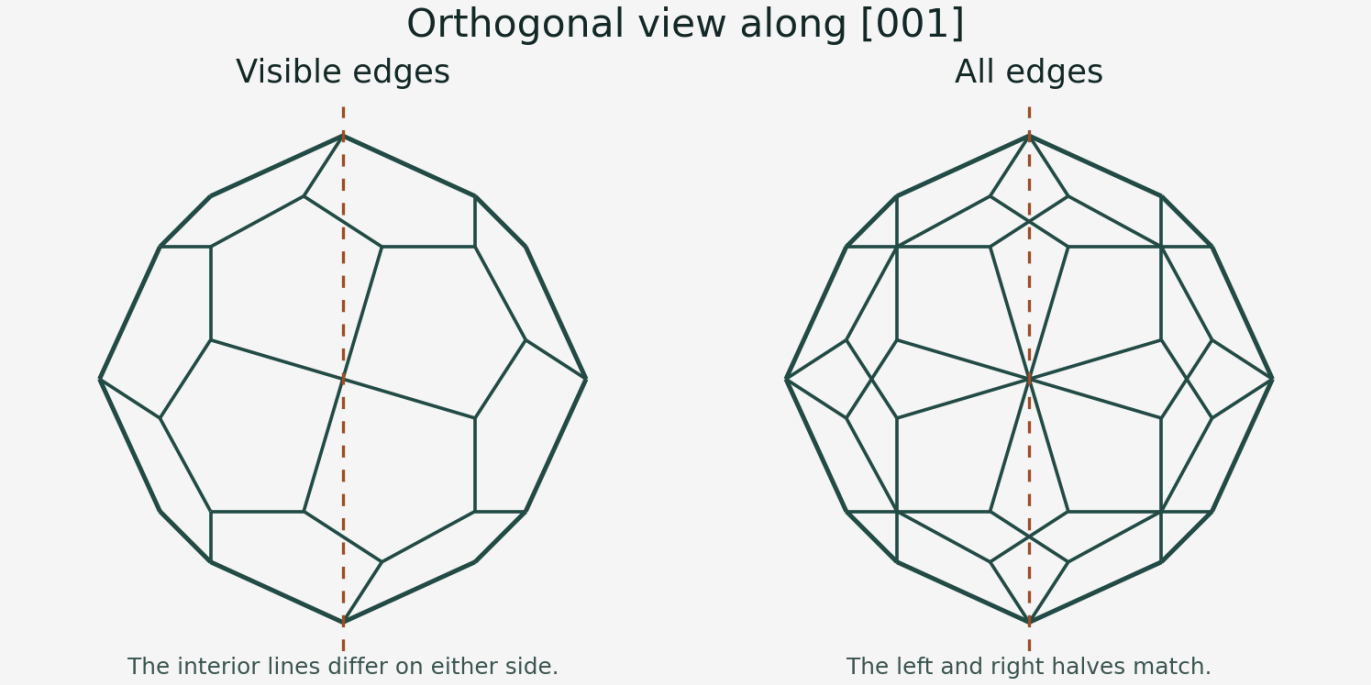
$$525.0523 + 0.5 \times 96.2241 + \pi \times 0.5^2 = 573.9498 \text{ mm}^2$$

Checking all viewing directions

The six planar faces divide the sphere of sightlines into 32 regions. Within each region the contributing faces and outline edges are fixed. Lower bounds for their area and projected lengths cover each region, including its boundaries. The stated direction attains the global bound. [2]

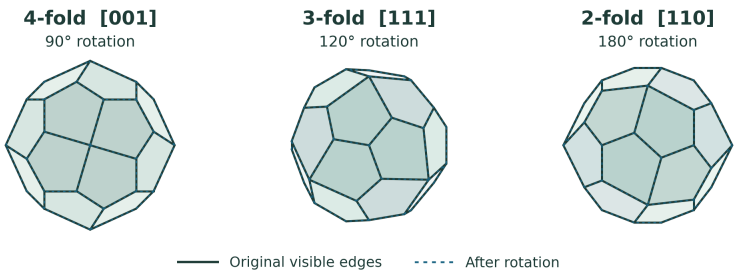
A second chiral solid

The pentagonal icositetrahedron illustrates the role of the drawing convention. In the [001] direction its outline and complete wireframe mirror across the dashed axis. The visible interior edges alone do not match across that line.



The two drawings share the same outer contour. Compare their interior edges across the dashed fold line. The brackets in [001] denote a direction in the model's coordinates.

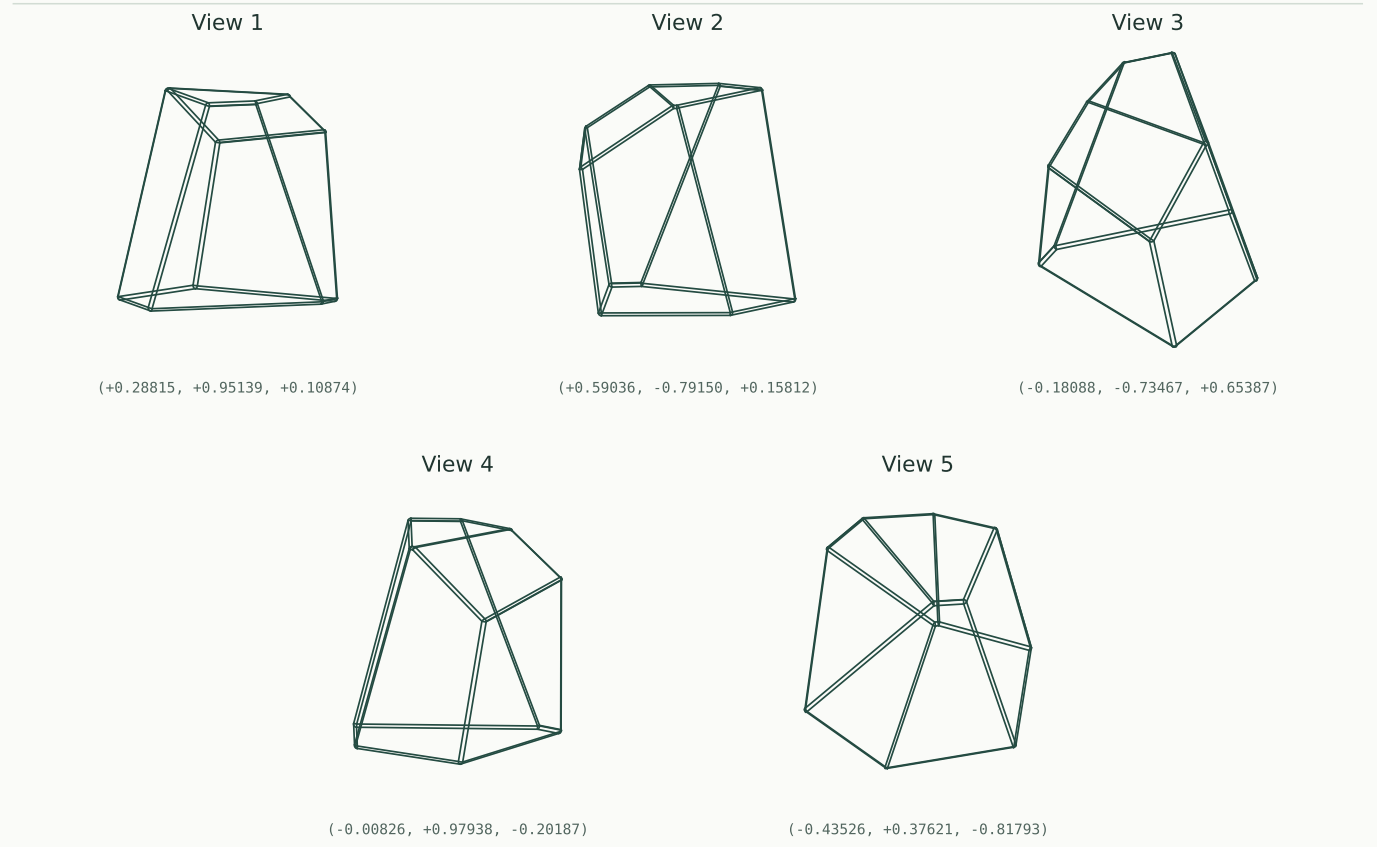
Three familiar rotational views



Along [001], [111] and [110], rotations of 90°, 120° and 180° respectively reproduce the projected pattern. These are three representative rotational views. The number of listed axes does not count the mirror-symmetric silhouettes. [3]

The eight-faced comparison model

The second supplied file, 8-faces.STEP, has eight planar faces with rounded transitions. Its 84 recorded edges consist of 39 straight segments and 45 circular arcs, whose supporting circles fall into 12 center groups.



Five sample wireframes from 8-faces.STEP. Each includes both front and back edges at the same drawing scale.

A separate exact certificate

The same center-pairing method excludes all 140,152 assignments for 12 separated centers, then checks the neighborhoods of all 66 pair directions. No orthographic view mirrors the complete recorded wireframe exactly. The silhouette alone would require its own test.

Reproducing the views

Directions below use the input STEP coordinate frame and point from object to camera. Screen up sets the rotation of the page. The fold passes through the projection of the corner-center mean, approximately (17.517359, 0, 0) mm.

The five-angle family

Let X = (1, 0, 0) and normalize the screen-up vector a = (0, 0.92046884, −0.39081596). At turn angle θ, use the camera direction n = X cos θ + (X × a) sin θ. Angles used in the figures are 0°, 41.52°, 80°, 105° and 130°.

Two directions to reproduce

Minimum silhouette • n = (0.74873017, 0.25906209, 0.61015569).
Full-wireframe near-match • n = (0.34777182, −0.93060115, −0.11417640), with screen up (0, 0.12177787, −0.99255738). Sampling all 51 curves at intervals up to 0.001 mm and adding half a sampling interval bounded the continuous mirror gap at 0.0078 mm.

Source models and geometry

Chiral_cube_D3_b_(more_skewed)_L_ScaleforDoublebowl-13JAN24.step and 8-faces.STEP. The chiral cube has six plane, twelve cylinder and eight sphere patches. Its transparent drawing uses the 51 edges explicitly recorded in the source file.

Further reading

[1] MIT Computer Graphics • Projection Transformations

[2] Hug and Schneider • Vectorial analogues of Cauchy's surface area formula

[3] Koca and Koca • Regular and Irregular Chiral Polyhedra, §5.1

Animations included in this PDF

Open the PDF's Attachments panel to save the 11-second see-through rotation (GIF) or the 76-second narrated explainer (MP4). The printed figures on pages 2–6 illustrate the same constructions.