The shadows of the Double Bowl model

A symmetry and minimum-area study of a triangulated solid

A shape can cast a shadow that looks evenly balanced even when its surface details fail the same fold test. This study measures both drawings for the supplied STL and checks how small its shadow can become.



The same pose in two views. The tilted transparent mesh is on the left. The camera records the filled silhouette on the right. The orange edge divides two triangles on one broad face.

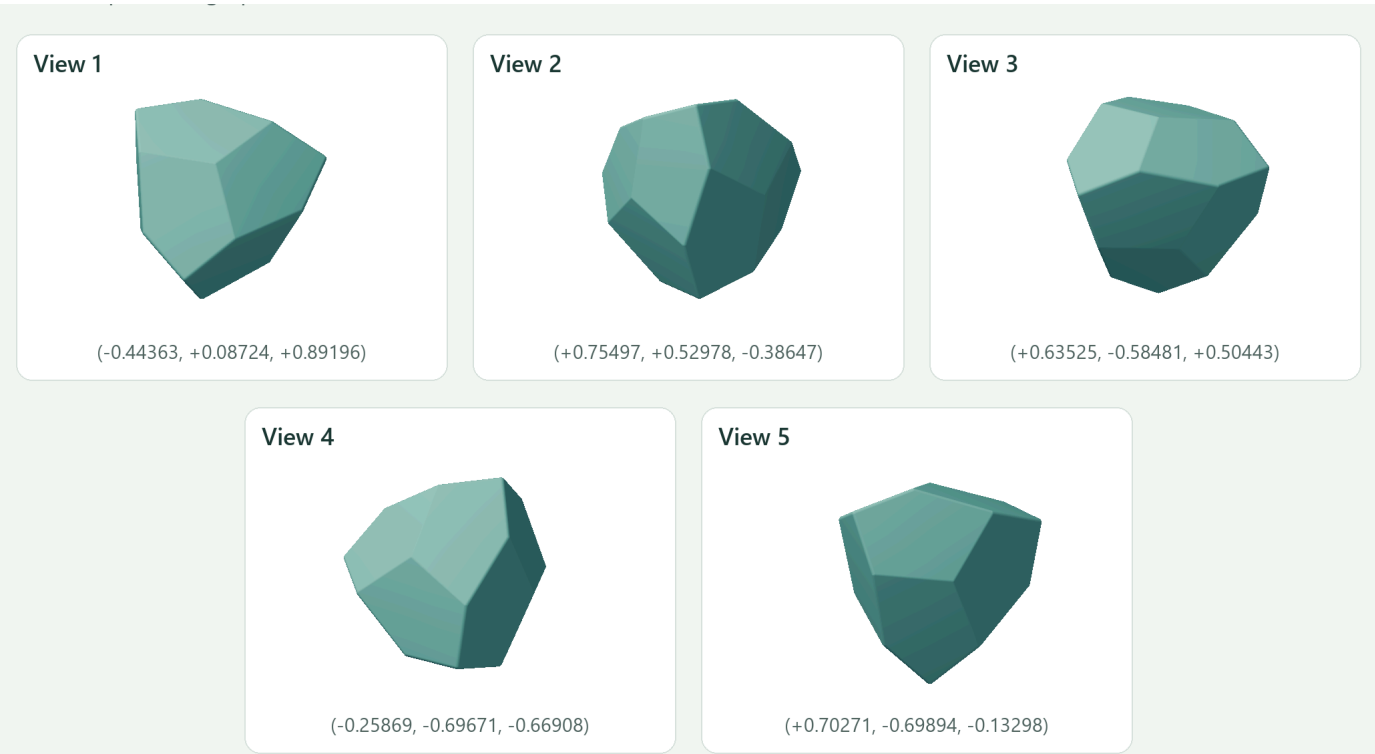
What the file shows

• Five different shadows match their reflections to within 0.0072 length unit about the tested vertical fold. Each still differs slightly from a perfect mirror.
• The 60 boundary segments of the broad faces match far more closely than the complete 2,709-edge triangle grid.
• The minimum projected area is approximately 615.7 square units. A view almost parallel to the Z axis reaches 615.70694.

STL stores coordinates without a physical unit. All reported lengths and areas use the file's coordinate scale. [1]

The surface in five random views

The binary STL contains one closed surface with 905 distinct vertices, 2,709 unique edges and 1,806 triangles. Every edge belongs to two oppositely oriented face boundaries. The coordinate spans along X, Y and Z are 30.83, 30.54 and 31.49 units.



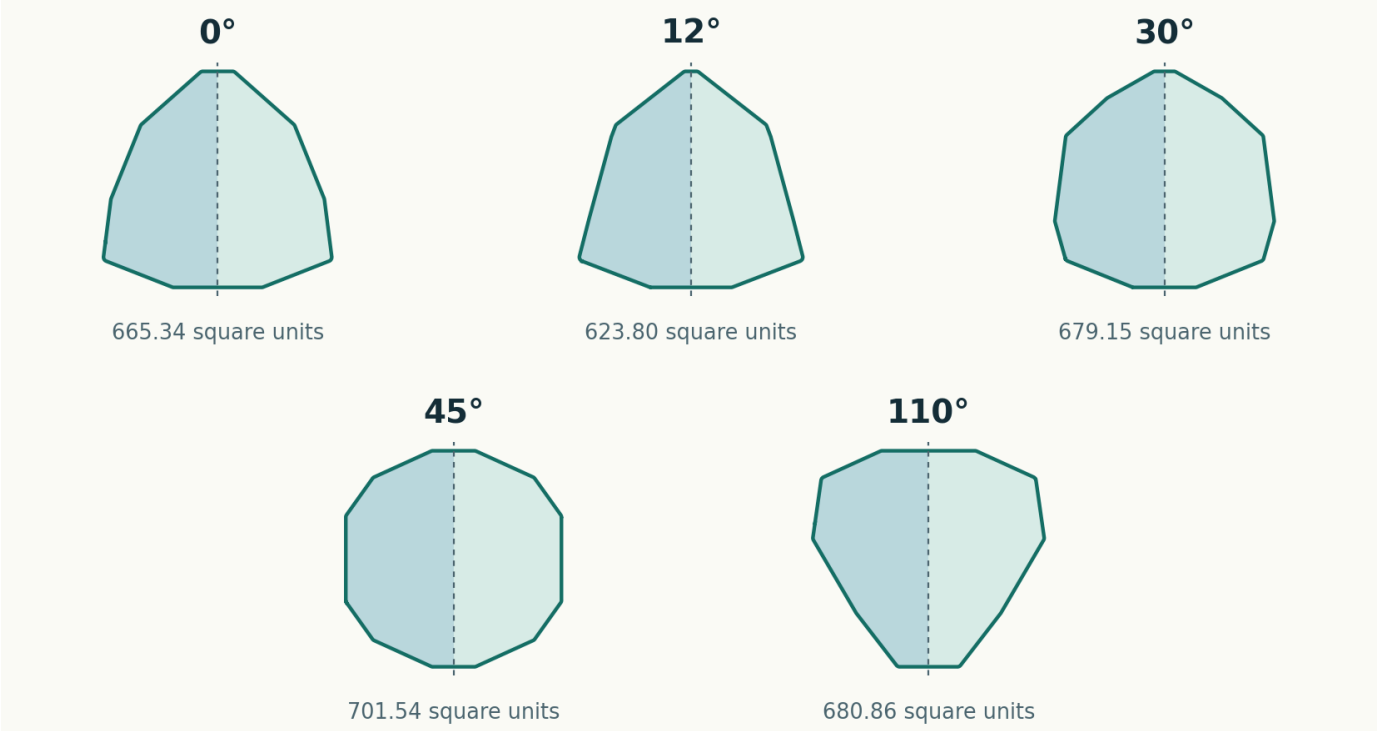
Five opaque orthographic views at one scale. The number triplets point from the object to the viewer. The randomly sampled directions have at least 27° of separation between viewing axes.

Twelve broad faces

The 36 largest triangles form twelve nearly planar pentagonal regions. Smaller triangles describe the rounded transitions between them. The enclosed volume is 10,539.63 cubic units, and the surface area is 2,640.97 square units.

Five near-mirror silhouettes

Hold one fitted half-turn axis upright and turn the camera around it. Reflect each silhouette across the vertical line through the projected volume centroid. The area changes with the camera angle, so these are five distinct shadows.



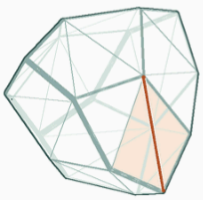
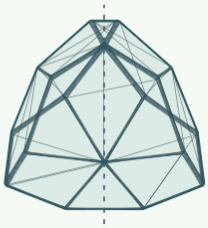
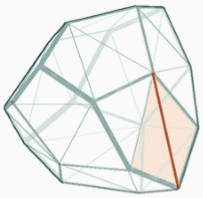
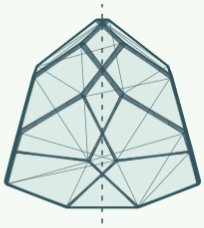
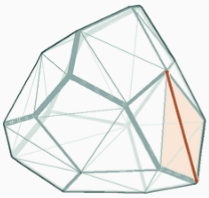
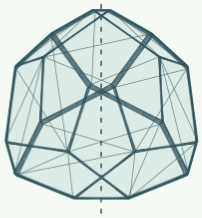
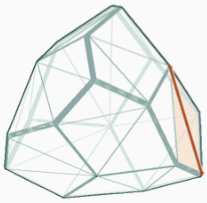
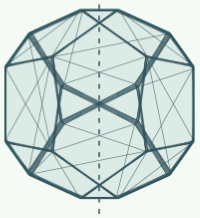
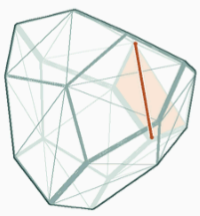
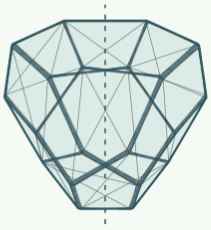
Measured mirror discrepancies

Camera turn	Area (units^2)	Outline gap ≤	Full grid gap
0°	665.34	0.0054	2.38 to 2.41
12°	623.80	0.0049	2.07 to 2.10
30°	679.15	0.0052	2.44 to 2.47
45°	701.54	0.0053	1.77 to 1.80
110°	680.86	0.0072	2.00 to 2.03

Gaps are in length units. They measure the greatest nearest-point separation after folding about the stated line. The comparison threshold of 0.01 unit is chosen for this study.

Following the five turns

The tilted model shows how the solid turns in space. The camera column shows the projected triangle grid, including edges on the back of the object.

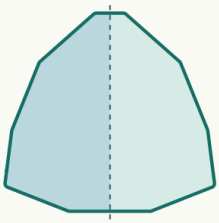
Turn	Tilted model	Camera view
0°		
12°		
30°		
45°		
110°		

The same five poses appear in the animated rotation embedded in this report.

Which parts of a drawing match?

An orthographic camera drops depth. A physical half-turn around a vertical axis reverses both left–right and front–back. Removing depth turns that operation into a reflection on the screen. The model approximates the half-turn with different precision in different features.

Solid outline



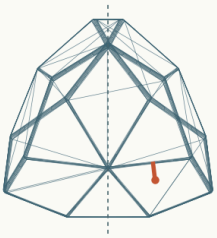
Outer edge of the filled projection

Major face boundaries



60 sides of 12 broad pentagons

Transparent mesh



2,709 unique triangle edges

At 0°, the contour deviates by about 0.005 unit. The orange segment in the grid joins a reflected point to its nearest original line, about 2.39 units away. A second experiment near +X reduced the full-grid gap to about 1.33 units.

What the half-turn predicts

The fitted rotations of the twelve major face normals have the pattern of tetrahedral rotation symmetry, including three approximate half-turn axes. The 60 sides of the broad faces can be paired under each half-turn with endpoint errors below 0.000072 unit. Every point along a paired side obeys the same bound after projection.

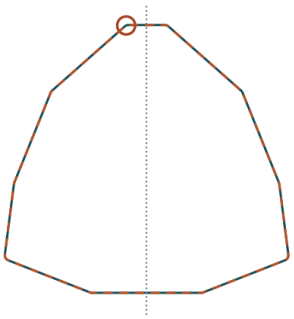
What survives a change of camera

For sightlines perpendicular to any of the three fitted half-turn axes, a conservative bound for the filled shadow and its outer contour is 0.025 unit. For the tighter 0.01 threshold, the five measurements on page 3 pass with room to spare. Small changes in these viewing directions also pass because projection varies continuously.

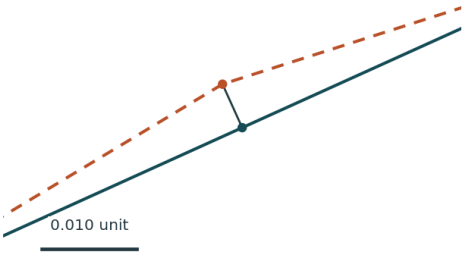
Where exact symmetry fails

At 0° the two sides look identical at ordinary drawing scale. The enlargement shows the original boundary in teal and its reflection across the tested vertical axis in dashed orange.

0° outline and reflection



Circled detail, enlarged



In the circled region, a point on the reflected outline misses the original by 0.00501 unit. The black bar gives the scale of the enlargement.

Could a different fold give a perfect match?

A mirror of the silhouette must also mirror its convex hull. At each of the five camera angles that hull has a unique longest side. A fold must preserve the side, and a fold along the side would send the hull's interior outside. The perpendicular bisector is the only candidate. A verified test places a reflected hull vertex outside the hull for every one of the five views.

The three-dimensional solid

The largest exactly planar patch in the recorded mesh is a unique scalene triangle. Any symmetry of the solid would have to fix all three of its corners. Only the identity or reflection in the triangle's plane could do this. That reflection sends part of the solid outside itself. The fitted rotations describe approximate symmetries of the physical features.

These five views have no exact fold. An exact mirror in some other projection of the silhouette or full triangle grid remains an open question for this file.

The smallest orthographic shadow

The minimum refers to the area enclosed by the filled shadow over every camera direction. The surface contains slight inward deviations from its convex hull. Using the hull as a substitute for the original STL would change the problem.



Opaque surface



Projected area 615.70694 square units

A near-minimum direction gives 615.70694 square units by taking the union of the projected triangles. Reversing the viewing direction gives the same area.

Bounding every orientation

For a lower bound, move selected supporting planes of the outer hull inward until their intersection forms a convex solid inside the STL. Every shadow of this inner solid fits inside the corresponding mesh shadow. Its least area can be found by comparing sightlines perpendicular to pairs of its supporting-plane normals.

For an upper bound, any actual view suffices. Combining the inner-solid calculation with the near-Z triangle union puts the global minimum between 615.6896 and 615.7071 square units numerically. Both ends round to 615.7.

The verifier checks the inner geometry with exact integers, evaluates 921,381 candidate directions with a numerical margin, and rounds a separate outer bound outward. The precise minimizing direction remains unresolved.

Coordinates and methods

The source file is T_a_L-100__Double_Bowl_Scale-3OCT22 (1).stl. View directions point from the object to the camera. The fold passes through the projection of the volume centroid C = (−533.840397, 263.654732, 865.755656).

Five-view camera path

Use screen up a = (0.68472164, 0.44462567, −0.57746367). Start at n_0 = (0.48431398, 0.31449047, 0.81641639). Normalize the vectors and turn by θ using $n(\theta) = n_0 \cos\theta + (n_0 \times a) \sin\theta$. The two other half-turn axes are approximately (0.04282778, −0.81552201, −0.57713919) and (−0.72754522, 0.37044820, −0.57744790).

Projection and accuracy

For each still, project the oriented triangles and take their planar union. Sample the resulting outline at intervals of at most 0.0006 unit. Compute the distance of each reflected sample to its nearest original segment. Adding half a sampling interval bounds the continuous gap. For the grid, both drawings were sampled at intervals of at most 0.02 unit.

The near-minimum camera is n ≈ (0.00000002, −0.00025789, −0.99999997). Independent triangle-union calculations agree to five decimal places. The smaller full-grid gap on page 5 uses n ≈ (0.99907021, 0.03209693, 0.02878370) with the second half-turn axis upright.

Scale and background

[1] Autodesk • STL units in exported models

Library of Congress • Binary STL file format

MIT Computer Graphics • Projection transformations

Embedded illustrations

The PDF’s Attachments panel contains the rotation GIF and the narrated explainer. The printed figures show the same model and can be read without playing either file.